Properties of perfect squares

Perfect squares end in 0, 1, 4, 5, 6, 9 only.

How to find the one's digit in the square of a number?

Example 1

- 65^2 = 65* 65.
- Multiply 5 *5 = 2<u>5</u>
- So, we can say that the ones place digit of the square of 65 will be 5.

Example 2

329^2 = 329 * 329

- Multiply 9 *9 = 8<u>1</u>
- So, we can say that the ones place digit of the square of 329 will be 1.

If a number has 0 in the unit's place, then it's square ends in 0.

Example 1

- 10^2 = 10 * 10 = 10<u>0</u>
- 430^2 = 430 * 430 = 184900

Important rule

Any number ending in 0 will have square ending in 0.

Any number ending in 2 or 8 will have square ending in 4.

Any number ending in 1 or 9 will have it's square ending in 1.

Any number ending in 4 or 6 will have it's square ending in 6.

Any number ending in 3 or 7 will have it's square ending in 9.

Any number ending in 5 will have it's square ending in 5.

Squares of even numbers are even and squares of odd numbers are odd.

Example 1: Even numbers

- 22^2 = 484
- 86^2 = 7396

Example 2: Odd numbers

- 81^2 = 6561
- 1001^2 = 1002001

A perfect square has even number of zeroes at the end.

Example 1

- 100 = 10 *10
- 100 is a perfect square and has 2 zeroes at the end which is an even number.

Example 2

- 1000= 100 *10

- 1000 is a non-perfect square as it has 3 (odd) zeroes at the end.

Example 3

- 3600 = 60 *60
- 3600 is a perfect square and has 2 (even) zeroes at the end.

The number of zeroes at the end of a perfect square is twice the number of zeroes at the end of the number.

If a number has 1 zero in the end, its square will have 2 zeroes. If a number has 2 zeroes in the end, it's square will have 2* 2 =4 zeroes in the end and so on.

We can say that if a number has n zeroes at the end, its square will have 2n zeroes at the end.

If a number is a perfect square, it has to be the sum of successive odd numbers starting from 1.

1 = 1
4 = 1+3
9 = 1 + 3+ 5
16 = 1 +3+ 5 + 7
25 = 1 + 3+ 5+ 7 +9
36 = 1+3+5+7 +9 +11
49 = 1 +3 + 5 + 7 + 9 + 11 + 13
64 = 1 +3 +5 + 7 + 9 +11 + 13 +15
81 = 1 +3 +5 + 7 + 9 +11 + 13 +15 +17
100 = 1 +3 +5 + 7 + 9 +11 + 13 +15 +17 +19

This means that the Sum of first n odd natural numbers is n^2

For example n = 5

First five natural odd numbers are 1 +3 +5 +7 +9 = 25 = 5^2.

Hence, proved.

We can say that if a natural number cannot be expressed as the sum of successive odd natural numbers starting from 1, then it is not a perfect square.